Eco-colonies

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Outline

1 Introduction to Eco-colonies
2 Unformal example
3 Definitions
4 Formal model
5 Generating power
   - Two types of eco-colonies
   - Eco-colonies and colonies
Origin of eco-colonies

- based on colonies with inspiration on eco-grammar systems,
Origin of eco-colonies

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- eco-colonies:
  - simple grammars (agents, components) – as in colonies,
  - self-developing environment – as in eco-grammar systems,
Origin of eco-colonies

- based on colonies with inspiration on eco-grammar systems,
- eco-colonies:
  - simple grammars (agents, components) – as in colonies,
  - self-developing environment – as in eco-grammar systems,
- two types:
  - 0L eco-colonies with one main alphabet (the environment is 0L-scheme) – as in eco-grammar systems,
  - E0L eco-colonies with two alphabets (a main and a terminal, the environment is E0L-scheme) – as in colonies.
Eco-colonies

Eco-colony as a grammar system

= model of a community of cooperating processes, grammar system
Eco-colonies

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- symbols – elements of the alphabet, objects,
Eco-colonies

Eco-colony as a grammar system

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- *symbols* – elements of the alphabet, objects,
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## Eco-colonies

### Eco-colony as a grammar system

= model of a community of cooperating processes, grammar system

- *symbols* – elements of the alphabet, objects,
- *environment* – contains symbols, the environment is self-developing,
- *word* – some of states of the environment,
Eco-colonies

**Eco-colony as a grammar system**

- = model of a community of cooperating processes, grammar system
  - *symbols* – elements of the alphabet, objects,
  - *environment* – contains symbols, the environment is self-developing,
  - *word* – some of states of the environment,
  - *agents (components)* – cooperating grammars, processes, subjects, working parallelly,
    - *start symbol* – what the agent can process, it looks for this symbol in the environment,
    - *language of the agent* (set of actions) – what the agent can do with its start symbol, the agent replaces it by some word of this language.
Example (motivation)

A

\(aBa, \ bb, \ cBc\)

B

\(AxA\)

A

\(aBa, \ \varepsilon\)

sensors, actuators

sensors, actuators

sensors, actuators

a A x A a d C C b

\(B \rightarrow bB\)  \(A \rightarrow A\)  \(C \rightarrow c\)  \(C \rightarrow cD\)  \(d \rightarrow d\)  \(b \rightarrow b\)  \(D \rightarrow C\)  \(a \rightarrow a\)  \(x \rightarrow x\)
Example (motivation)

\[ aBa, bb, cBc \]

\[ AxA \]

\[ aBa, \varepsilon \]

**Found**

**Action**

**Sensors, actuators**

<table>
<thead>
<tr>
<th>a</th>
<th>A</th>
<th>x</th>
<th>A</th>
<th>a</th>
<th>d</th>
<th>C</th>
<th>C</th>
<th>b</th>
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</table>

\[ B \rightarrow bB \quad A \rightarrow A \quad C \rightarrow c \quad C \rightarrow cD \quad d \rightarrow d \]

\[ x \rightarrow x \quad c \rightarrow c \quad C \rightarrow cD \quad b \rightarrow b \quad D \rightarrow C \quad a \rightarrow a \]
Example (motivation)

A

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sensors, actuators

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found

sensors, actuators

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\[ aBa, \varepsilon \]

action

found

sensors, actuators

\[ a \mid A \mid x \mid A \mid a \mid d \mid C \mid C \mid b \]

\[
B \to bB \quad A \to A \\
C \to c \\
c \to c \\
C \to cD \\
d \to d \\
b \to b \\
a \to a \\
D \to C
\]
Example (motivation)

- $aBa, bb, cBc$
- $AxA$
- $aBa, \varepsilon$

<table>
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<tr>
<th>$a$</th>
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<th>$x$</th>
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- $B \rightarrow bB$
- $A \rightarrow A$
- $C \rightarrow c$
- $C \rightarrow cD$
- $d \rightarrow d$
- $x \rightarrow x$
- $c \rightarrow c$
- $b \rightarrow b$
- $a \rightarrow a$
- $D \rightarrow C$
Example (motivation)

\[ \text{found} \quad \text{action} \quad \text{found} \quad \text{action} \quad \text{found} \quad \text{action} \]

- Sensors, actuators
- \( aBa, bb, cBc \)
- \( AxA \)
- \( aBa, \varepsilon \)

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- \( c \rightarrow c \)
- \( b \rightarrow b \)
- \( d \rightarrow d \)
- \( a \rightarrow a \)
- \( D \rightarrow C \)
Example (motivation)

- A: found action, $aBa$, $bb$, $cBc$
- B: found action, $AxA$
- A: found action, $aBa$, $\varepsilon$

<table>
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<th>a</th>
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<th>B</th>
<th>a</th>
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- $B \rightarrow bB$
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- $D \rightarrow C$
Example (motivation)

A

\[ aBa, \ bb, \ cBc \]

found

sensors, actuators

action

B

AxA

found

sensors, actuators

action

A

\[ aBa, \ \varepsilon \]

found

sensors, actuators

action

\[
\begin{align*}
A & \rightarrow A \\
B & \rightarrow bB \\
x & \rightarrow x \\
c & \rightarrow c \\
C & \rightarrow c \\
C & \rightarrow cD \\
d & \rightarrow d \\
b & \rightarrow b \\
D & \rightarrow C
\end{align*}
\]
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Unformal example

Example (motivation)

A \rightarrow aBa, bb, cBc

action

found

sensors, actuators

B \rightarrow AxA

action

found

sensors, actuators

A \rightarrow aBa, \varepsilon

action

found

sensors, actuators

a \ x \ c \ B \ a \ d \ c \ D \ c \ D \ b \ b

B \rightarrow bB

A \rightarrow A

c \rightarrow c

C \rightarrow c \ D

a \rightarrow a

b \rightarrow b

D \rightarrow C

d \rightarrow d
Example (motivation)

\[
\begin{align*}
A & \rightarrow A \\
X & \rightarrow X \\
C & \rightarrow c \\
C & \rightarrow cD \\
a & \rightarrow a \\
b & \rightarrow b \\
d & \rightarrow d \\
B & \rightarrow bB \\
C & \rightarrow c \\
C & \rightarrow cD \\
A & \rightarrow A \\
x & \rightarrow x \\
D & \rightarrow C
\end{align*}
\]
Example (formal model)

Agents:

\[ A_1 = (A, \{aBa, bb, cBc\}) \]
\[ A_2 = (B, \{AxA\}) \]
\[ A_3 = (A, \{aBa, \varepsilon\}) \]
## Example (formal model)

**Agents:**

\[ A_1 = (A, \{ aBa, bb, cBc \}) \]
\[ A_2 = (B, \{ AxA \}) \]
\[ A_3 = (A, \{ aBa, \varepsilon \}) \]

**Rules in the environment:**

\[ C \rightarrow cD \]
\[ D \rightarrow C \]
\[ A \rightarrow A \]
\[ d \rightarrow d \]
\[ C \rightarrow c \]
\[ B \rightarrow bB \]
\[ a \rightarrow a \]
\[ x \rightarrow x \]
\[ b \rightarrow bb \]
\[ c \rightarrow c \]
Example (formal model)

<table>
<thead>
<tr>
<th>Agents:</th>
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<tbody>
<tr>
<td>$A_1 = (A, {aBa, bb, cBc})$</td>
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</tr>
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<td>$C \rightarrow cD$</td>
<td>$D \rightarrow C$</td>
</tr>
<tr>
<td>$C \rightarrow c$</td>
<td>$B \rightarrow bB$</td>
</tr>
<tr>
<td>$b \rightarrow bb$</td>
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<th>Alphabets:</th>
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</thead>
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<tr>
<td>Alphabet of the environment: $V = {A, B, C, D, a, b, c, d, x}$</td>
<td></td>
</tr>
<tr>
<td>Terminal alphabet: $T = {a, b, c, d, x}$</td>
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</tbody>
</table>
Definitions of eco-colonies

Definition (E0L eco-colony) of degree \( n \), \( n \geq 1 \), is an \((n + 2)\)-tuple \( \Sigma = (E, A_1, A_2, \ldots, A_n, w_0) \), where

- \( E = (V, T, P) \) is E0L scheme, where
  - \( V \) is a finite non-empty alphabet,
  - \( T \) is a non-empty terminal alphabet, \( T \subseteq V \),
  - \( P \) is a finite set of E0L rewriting rules over \( V \),
- \( A_i = (S_i, F_i) \), \( 1 \leq i \leq n \), is the \( i \)-th agent, where
  - \( S_i \in V \) is the start symbol of the agent,
  - \( F_i \subseteq (V - \{S_i\})^* \) is a finite set of action rules of the agent (the language of the agent),
- \( w_0 \) is the axiom.
Definitions of eco-colonies

Definition (0L eco-colony)
of degree \( n, n \geq 1 \), is an \((n + 2)\)-tuple \( \Sigma = (E, A_1, A_2, \ldots, A_n, w_0) \), where
- \( E = (V, P) \) is 0L scheme, where
  - \( V \) is a finite non-empty alphabet,
  - \( (T = V, \text{not used ind this definition}) \),
  - \( P \) is a finite set of 0L rewriting rules over \( V \),
- \( A_i = (S_i, F_i), 1 \leq i \leq n \), is the \( i \)-th agent, where
  - \( S_i \in V \) is the start symbol of the agent,
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    (the language of the agent),
- \( w_0 \) is the axiom.
Definitions of derivation steps

Definition (Weakly competitive parallel derivation step wp)

in an eco-colony $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$ is $\alpha \xrightarrow{\text{wp}} \beta$, where

- $\alpha = \gamma_0 S_{i_1} \gamma_1 S_{i_2} \gamma_2 \ldots \gamma_{r-1} S_{i_r} \gamma_r$, $r > 0$,
- $\beta = \gamma'_0 f_{i_1} \gamma'_1 f_{i_2} \gamma'_2 \ldots \gamma'_{r-1} f_{i_r} \gamma'_r$, $A_{i_k} = (S_{i_k}, F_{i_k})$, $f_{i_k} \in F_{i_k}$, $1 \leq k \leq r$ (the agent $A_{i_k}$ is active in this derivation step),
- $\{i_1, i_2, \ldots, i_r\} \subseteq \{1, 2, \ldots, n\}$, $i_k \neq i_m$ for every $k \neq m$, $1 \leq k, m \leq r$,
- for every symbol $S \in V$ if $|\gamma_0 \gamma_1 \ldots \gamma_r|_S > 0$ then every agent with the start symbol $S$ must be active (if agents can work they must work),
- $\gamma_k \xrightarrow{E} \gamma'_k$, $\gamma_k \in V^*$, $0 \leq k \leq r$, is the derivation step of the scheme $E$. 
Definitions of derivation steps

Definition (derivation step \( ap \) – all are working parallely)

In an eco-colony \( \Sigma = (E, A_1, A_2, \ldots, A_n, w_0) \) is \( \alpha \xrightarrow{ap} \beta \), where

- \( \alpha = \gamma_0 S_{i_1} \gamma_1 S_{i_2} \gamma_2 \cdots \gamma_{n-1} S_{i_n} \gamma_n \),
- \( \beta = \gamma'_0 f_{i_1} \gamma'_1 f_{i_2} \gamma'_2 \cdots \gamma'_{n-1} f_{i_n} \gamma'_n \), \( A_{i_k} = (S_{i_k}, F_{i_k}) \), \( f_{i_k} \in F_{i_k} \), \( 1 \leq k \leq n \),
- \( \{i_1, i_2, \ldots, i_n\} = \{1, 2, \ldots, n\} \) (every agent works in every derivation step),
- \( \gamma_k \xrightarrow{E} \gamma'_k \), \( \gamma_k \in V^* \), \( 0 \leq k \leq n \), is the derivation step of the scheme \( E \).
Definition of language

Definition

Let $\Sigma$ be an 0L eco-colony, $\Sigma = (E, A_1, A_2, \ldots, A_n, w_0)$. The language generated by the type of derivation $x$, $x \in \{wp, ap\}$ in $\Sigma$ is

$$L(\Sigma, x) = \{ w \in V^* \mid w_0 \xrightarrow{\sigma}^* w \}$$

Definition

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$$L(\Sigma, x) = \{ w \in T^* \mid w_0 \xrightarrow{\sigma}^* w \}$$
Example

\[ \Sigma = (E, A_1, A_2, AbB), \text{ where} \]
\[ E = (\{A, B, a, b\}, \{a, b\}, \{a \rightarrow a, b \rightarrow bb\}), \]
\[ A_1 = (A, \{aB, \varepsilon\}), \]
\[ A_2 = (B, \{aA, \varepsilon\}) \]
Example

\[ \Sigma = (E, A_1, A_2, AbB), \text{ where} \]
\[ E = (\{A, B, a, b\}, \{a, b\}, \{a \rightarrow a, b \rightarrow bb\}), \]
\[ A_1 = (A, \{aB, \varepsilon\}), \]
\[ A_2 = (B, \{aA, \varepsilon\}) \]

\[ AbB \xrightarrow{ap} aBb^2aA \xrightarrow{ap} a^2Ab^4a^2B \xrightarrow{ap} a^3Bb^8a^3A \xrightarrow{ap} \]
\[ \xrightarrow{ap} a^4Ab^{16}a^4B \xrightarrow{ap} a^5Bb^{36}a^5A \xrightarrow{ap} \ldots \]

\[ AbB \xrightarrow{wp} aBb^2aA \xrightarrow{wp} a^2Ab^4a^2B \xrightarrow{wp} a^2b^8a^3A \xrightarrow{wp} a^2b^{16}a^4B \xrightarrow{wp} \]
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Example

\( \Sigma = (E, A_1, A_2, AbB) \), where
\( E = (\{A, B, a, b\}, \{a, b\}, \{a \rightarrow a, b \rightarrow bb\}) \),
\( A_1 = (A, \{aB, \varepsilon\}) \),
\( A_2 = (B, \{aA, \varepsilon\}) \)

\( AbB \xrightarrow{ap} aBb^2 aA \xrightarrow{ap} a^2 Ab^4 a^2 B \xrightarrow{ap} a^3 Bb^8 a^3 A \xrightarrow{ap} \)
\( \xrightarrow{ap} a^4 Ab^{16} a^4 B \xrightarrow{ap} a^5 Bb^{36} a^5 A \xrightarrow{ap} \ldots \)

\( AbB \xrightarrow{wp} aBb^2 aA \xrightarrow{wp} a^2 Ab^4 a^2 B \xrightarrow{wp} a^2 b^8 a^3 A \xrightarrow{wp} a^2 b^{16} a^4 B \xrightarrow{wp} \)
\( \xrightarrow{wp} a^2 b^{36} a^5 B \xrightarrow{wp} \ldots \)

Generated languages:
\( L(\Sigma, ap) = \left\{ a^n b^{2^n} a^n \mid n \geq 0 \right\} \)
\( L(\Sigma, wp) = \left\{ a^i b^{2^n} a^j \mid n \geq 0, \ 0 \leq i, j \leq n \right\} \)
Notation

### \( 0EC_x \)
\( x \in \{wp, ap\} \) class of 0L eco-colonies with \( x \) type of derivation

### \( EEC_x \)
\( x \in \{wp, ap\} \) class of E0L eco-colonies with \( x \) type of derivation

### \( COL_x \)
\( x \in \{b, t, wp, sp\} \) class of colonies with the \( x \) type of derivation

### \( COL_T^x \)
\( x \in \{b, t, wp, sp\} \) class of colonies with the \( x \) type of derivation, \( V = T \)
Results proved in the paper

Theorem

\[ 0 \mathcal{E}_{C_{ap}} \subset \mathcal{E}_{E_{C_{ap}}} \] (1)
0EC_{ap} \subset EEC_{ap}.

The relation 0EC_{ap} \subseteq EEC_{ap} is trivial, 0L eco-colonies are special type of E0L eco-colonies (for V = T).

\[ L_1 = \{ a^{2^n} \mid n \geq 1 \} \]

This language is generated by the E0L eco-colony \( \Sigma = (E, A_1, A_2, UVa) \), where
\[ E = (\{a, U, V\}, \{a\}, \{a \rightarrow aa, U \rightarrow U, V \rightarrow V\}) \],
\[ A_1 = (U, \{V, \varepsilon\}) \],
\[ A_2 = (V, \{U, \varepsilon\}) \].

\[ UVa \xrightarrow{ap} VUa^2 \xrightarrow{ap} UVa^4 \xrightarrow{ap} VUa^8 \xrightarrow{ap} UVa^{16} \xrightarrow{ap} \ldots \xrightarrow{ap} a^{2^n} \]

L_1 is not generated by any 0L eco-colony with ap derivation (see the proof of Theorem 1 in the paper).
Results proved in the paper

**Theorem**

\[ xEC_y - COL_z \neq \emptyset \]  \hspace{1cm} (2)

where \( x \in \{0, E\} \), \( y \in \{wp, ap\} \), \( z \in \{b, t, wp, sp\} \).
Results proved in the paper

\( xEC_y \neq COL_z \).

We use the language

\[
L_2 = \left\{ cd a^{2n} b^{2n} \mid n \geq 0 \right\} \cup \left\{ dca^{2n+1} b^{2n+1} \mid n \geq 0 \right\}
\]

This language can be generated by the eco-colony (ap as well as wp) \( \Sigma = (E, A_1, A_2, cda) \), where \( E = (\{a, b, c, d\}, \{a \rightarrow aa, b \rightarrow bb, c \rightarrow c, d \rightarrow d\}) \), \( A_1 = (c, \{d\}) \), \( A_2 = (d, \{c\}) \).

\[
cdab \Rightarrow dca^2b^2 \Rightarrow cda^4b^4 \Rightarrow dca^8b^8 \Rightarrow cda^{16}b^{16} \Rightarrow \ldots
\]

\( L_2 \) is not generated by any colony with \( b, t, wp, sp \) derivation (see the proof of Theorem 2 in the paper).
Results proved in the paper

**Theorem**

\[ COL_x - 0EC_{wp} \neq \emptyset, \quad x \in \{b, t, wp, sp\} \]  

(3)
Results proved in the paper

\[
\text{COL}_x - 0\text{EC}_{wp} \neq \emptyset.
\]

The language

\[
L_3 = \{ a^{15-2n} b^n c b^n d \mid 0 \leq n < 7, n \text{ is even} \} \\
\cup \{ a^{15-2n} b^n d b^n c \mid 0 < n \leq 7, n \text{ is odd} \}
\]

is a finite language, so \( L_3 \in \text{COL}_x \) for \( x \in \{b, t, wp, sp\} \).

This language is not in \( 0\text{EC}_{wp} \) (see the proof of Theorem 3 in the paper).
Results proved in the paper

**Theorem**

\[ COL_x - 0EC_{ap} \neq \emptyset, \quad x \in \{b, t, wp, sp\} \] (4)
Results proved in the paper

\[ \text{COL}_x - 0\text{EC}_{ap} \neq \emptyset. \]

The language

\[ L_4 = \{a, aa\} \]

is not generated by any 0L eco-colony. But this language is finite, so \( L_4 \in \text{COL}_x \) for \( x \in \{b, t, wp, sp\} \) (see the proof of Theorem 4 in the paper).
Results proved in the paper

Theorem

\[ \text{COL}_b \subset \text{EEC}_{ap} \] (5)
Theorem

\[ \text{COL}_b \subseteq \text{EEC}_{ap} \]  \hspace{1cm} (5)

The proof can be found in the paper.
We demonstrate the construction of the proof on the colony generating this language:

\[ L_5 = \left\{ waw^R a^i \mid w \in \{0, 1\}^*, \ i > 0 \right\} \]
Example: $L_5 = \{waw^Ra^i \mid w \in \{0, 1\}^*, \; i > 0\}$

Colony: $\mathcal{C} = (\{S, H, H', A, A', 0, 1, a\}, \{0, 1, a\}, \mathcal{R}, S)$
the set of components $\mathcal{R}$ is
$\mathcal{R} = \{ (S, \{HA\}), (H, \{0H'0, 1H'1, a\}), (A, \{aA', a\}), (H', \{H\}), (A', \{A\}) \}
Example: \( L_5 = \{ w a^i w^R | w \in \{0,1\}^*, \ i > 0 \} \)

Colony: \( C = (\{S, H, H', A, A', 0, 1, a\}, \{0, 1, a\}, \mathcal{R}, S) \)
the set of components \( \mathcal{R} \) is 
\( \mathcal{R} = \{ (S, \{HA\}), (H, \{0H'0, 1H'1, a\}), (A, \{aA', a\}), 
(H', \{H\}), (A', \{A\}) \} \)

Agens work but do not make the word (longer axiom),
components \( \Rightarrow \) rules in the environment.

E0L eco-colony with ap derivation: \( \Sigma = (E, A_1, A_2, BCS) \),
\( E = (\{B, C, S, H, H', A, A', 0, 1, a\}, \{0, 1, a\}, P) \),
\( A_1 = (B, \{C, \varepsilon\}) \), \( A_2 = (C, \{B, \varepsilon\}) \),
the set of rules \( P \) in the environment is 
\( P = \{ H \rightarrow H|0H'0|1H'1|A, \ H' \rightarrow H'|H, \ 1 \rightarrow 1, \ a \rightarrow a, 
A \rightarrow A|aA'|a, \ A' \rightarrow A'|A, \ 0 \rightarrow 0, \ S \rightarrow S|HA \} \).
Results proved in the paper

Corollaries

1. \( xEC_y - COL_T^z \neq \emptyset \)
   where \( x \in \{0, E\} \), \( y \in \{wp, ap\} \), \( z \in \{b, t, wp, sp\} \).

2. The set of languages \( 0EC_{wp} \) is incomparable to the sets of languages \( COL_b, COL_t, COL_{wp} \) and \( COL_{sp} \).

3. \( COL_b^T \subset EEC_{wp} \), \( COL_{wp}^T \subset EEC_{wp} \)

4. \( COL_b^T \subset EEC_{ap} \)

5. \( COL_b^T \subset EEC_{ap} \)

Follow from the previous results.
Thank you for your attention.