Eco-colonies

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Outline

Origin of eco-colonies

- based on colonies with inspiration on eco-grammar systems,

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- eco-colonies:
  - simple grammars (agents, components) – as in colonies,
  - self-developing environment – as in eco-grammar systems,

Origin of eco-colonies

- based on colonies with inspiration on eco-grammar systems,
- eco-colonies:
  - simple grammars (agents, components) – as in colonies,
  - self-developing environment – as in eco-grammar systems,
- two types:
  - 0L eco-colonies with one main alphabet (the environment is 0L-scheme) – as in eco-grammar systems,
  - E0L eco-colonies with two alphabets (a main and a terminal, the environment is E0L-scheme) – as in colonies.

Eco-colonies

Eco-colony as a grammar system

= model of a community of cooperating processes, grammar system

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- *word* – some of states of the environment,

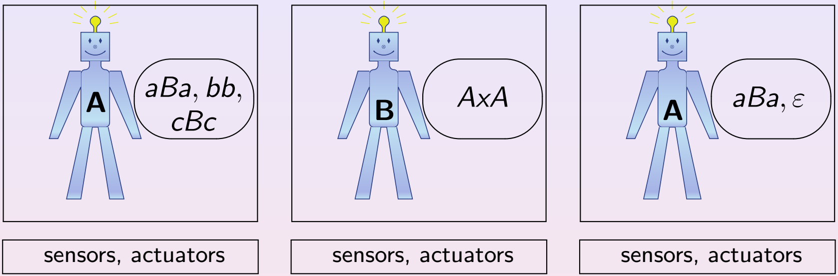
Eco-colonies

Eco-colony as a grammar system

= model of a community of cooperating processes, grammar system

- *symbols* – elements of the alphabet, objects,
- *environment* – contains symbols, the environment is self-developing,
- *word* – some of states of the environment,
- *agents (components)* – cooperating grammars, processes, subjects, working parallely,
  - *start symbol* – what the agent can process, it looks for this symbol in the environment,
  - *language of the agent* (set of actions) – what the agent can do with its start symbol, the agent replaces it by some word of this language.

Example (motivation)



a	A	x	A	a	d	C	C	b							

$B \to bB$ $\quad A \to A$ $\quad C \to c$ $\quad c \to c$ $\quad C \to cD$ $\quad d \to d$ $\quad b \to b$

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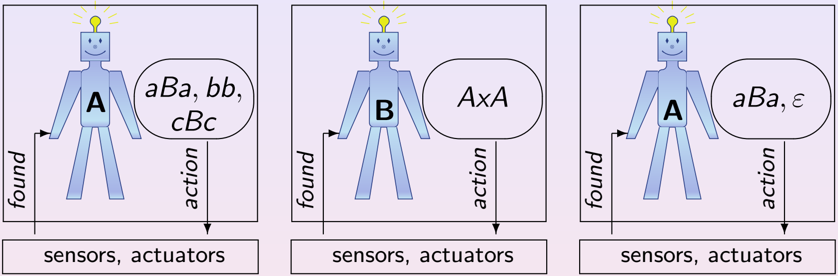

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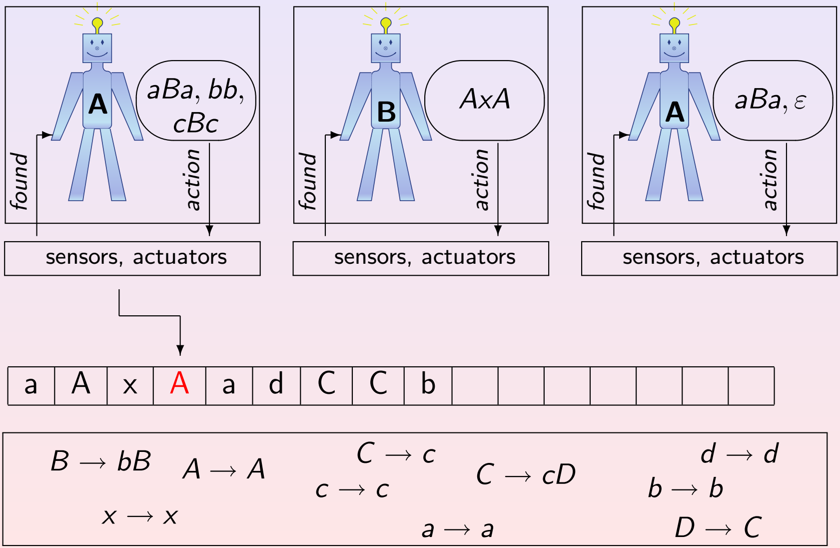

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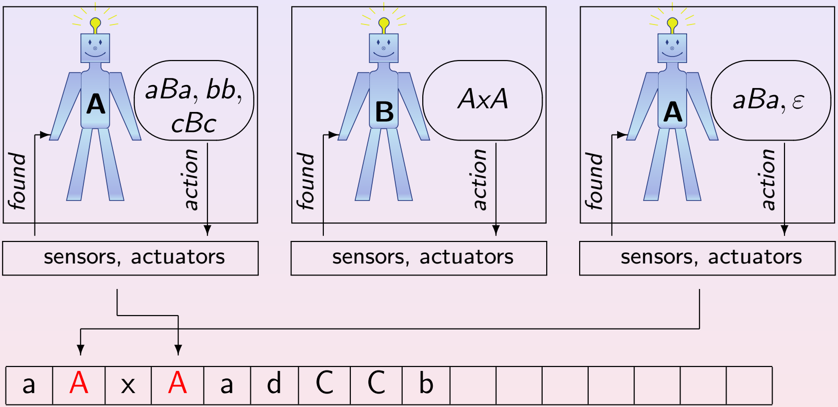

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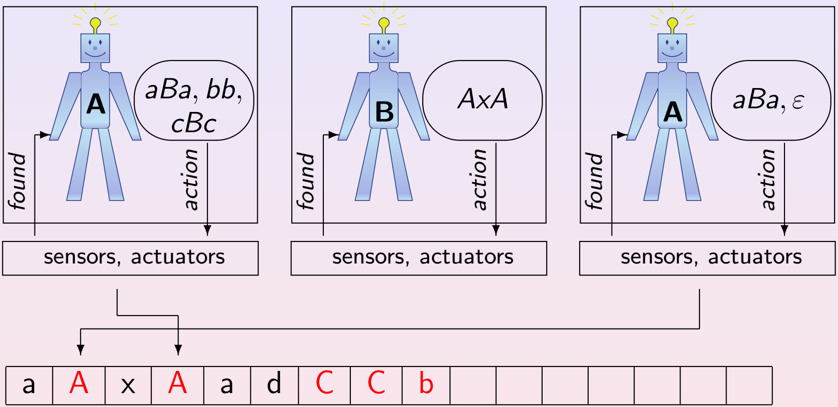

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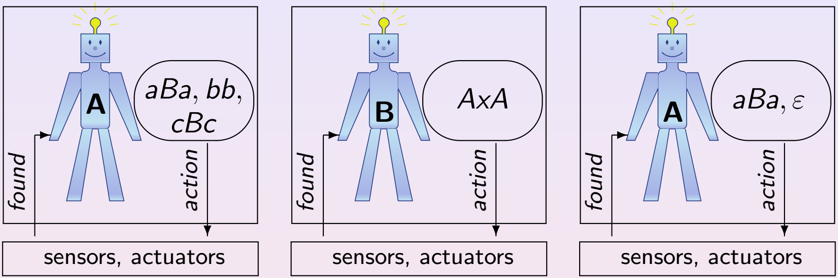

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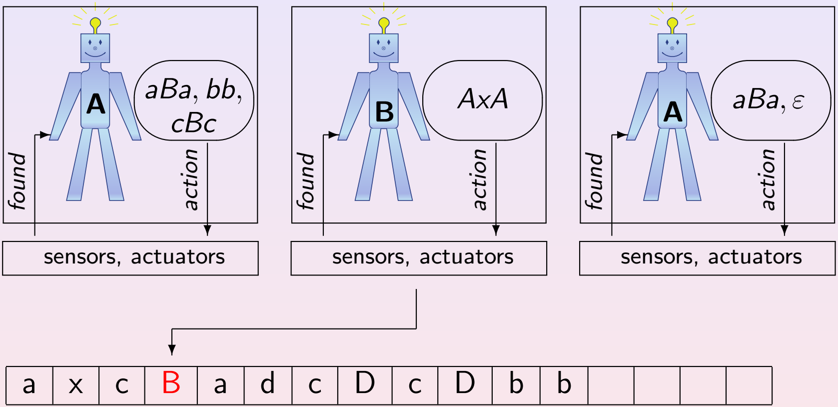

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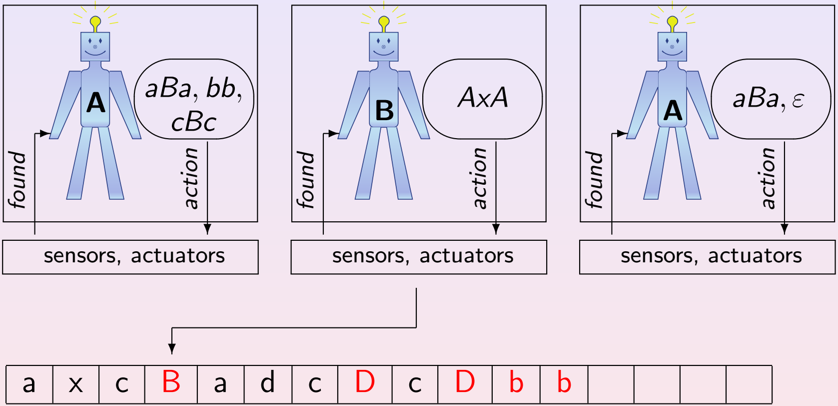

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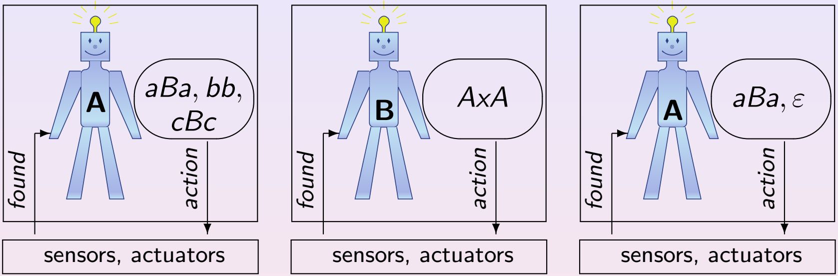

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Example (formal model)

Agents:

$A_1 = (A,\ \{aBa,\ bb,\ cBc\})$
$A_2 = (B,\ \{AxA\})$
$A_3 = (A,\ \{aBa,\ \varepsilon\}$

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Rules in the environment:

$C \rightarrow cD$	$D \rightarrow C$	$A \rightarrow A$	$d \rightarrow d$
$C \rightarrow c$	$B \rightarrow bB$	$a \rightarrow a$	$x \rightarrow x$
$b \rightarrow bb$		$c \rightarrow c$	

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Rules in the environment:

$C \longrightarrow cD$	$D \longrightarrow C$	$A \longrightarrow A$	$d \longrightarrow d$
$C \longrightarrow c$	$B \longrightarrow bB$	$a \longrightarrow a$	$x \longrightarrow x$
$b \longrightarrow bb$		$c \longrightarrow c$	

Alphabets:

Alphabet of the environment: $V = \{A, B, C, D, a, b, c, d, x\}$
Terminal alphabet: $T = \{a, b, c, d, x\}$

Definitions of eco-colonies

Definition (E0L eco-colony)

of degree n, $n \geq 1$, is an $(n+2)$-tuple
$\Sigma = (E, A_1, A_2, \dots, A_n, w_0)$, where

- $E = (V, T, P)$ is E0L scheme, where
  - V is a finite non-empty alphabet,
  - T is a non-empty terminal alphabet, $T \subseteq V$,
  - P is a finite set of E0L rewriting rules over V,
- $A_i = (S_i, F_i)$, $1 \leq i \leq n$, is the i-th agent, where
  - $S_i \in V$ is the start symbol of the agent,
  - $F_i \subseteq (V - \{S_i\})^*$ is a finite set of action rules of the agent (the language of the agent),
- w_0 is the axiom.

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$\Sigma = (E, A_1, A_2, \dots, A_n, w_0)$, where

- $E = (V, P)$ is 0L scheme, where
  - V is a finite non-empty alphabet,
  - ($T = V$, *not used ind this definition*),
  - P is a finite set of 0L rewriting rules over V,
- $A_i = (S_i, F_i)$, $1 \leq i \leq n$, is the i-th agent, where
  - $S_i \in V$ is the start symbol of the agent,
  - $F_i \subseteq (V - \{S_i\})^*$ is a finite set of action rules of the agent (the language of the agent),
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Definitions of derivation steps

Definition (Weakly competitive parallel derivation step *wp*)

in an eco-colony $\Sigma = (E, A_1, A_2, \dots, A_n, w_0)$ is $\alpha \overset{wp}{\Longrightarrow} \beta$, where

- $\alpha = \gamma_0 S_{i_1} \gamma_1 S_{i_2} \gamma_2 \dots \gamma_{r-1} S_{i_r} \gamma_r,\ \ r > 0$,
- $\beta = \gamma'_0 f_{i_1} \gamma'_1 f_{i_2} \gamma'_2 \dots \gamma'_{r-1} f_{i_r} \gamma'_r, \quad A_{i_k} = (S_{i_k}, F_{i_k}),\ \ f_{i_k} \in F_{i_k}$, $1 \leq k \leq r$ (the agent A_{i_k} is *active* in this derivation step),
- $\{i_1, i_2, \dots, i_r\} \subseteq \{1, 2, \dots, n\}$, $i_k \neq i_m$ for every $k \neq m,\ 1 \leq k, m \leq r$,
- for every symbol $S \in V$ if $|\gamma_0 \gamma_1 \dots \gamma_r|_S > 0$ then every agent with the start symbol S must be active (if agents can work they must work),
- $\gamma_k \overset{E}{\Longrightarrow} \gamma'_k,\ \ \gamma_k \in V^*,\ \ 0 \leq k \leq r$, is the derivation step of the scheme E.

Definitions of derivation steps

Definition (derivation step *ap* – all are working parallely)

in an eco-colony $\Sigma = (E, A_1, A_2, \dots, A_n, w_0)$ is $\alpha \stackrel{ap}{\Longrightarrow} \beta$, where

- $\alpha = \gamma_0 S_{i_1} \gamma_1 S_{i_2} \gamma_2 \dots \gamma_{n-1} S_{i_n} \gamma_n$,
- $\beta = \gamma'_0 f_{i_1} \gamma'_1 f_{i_2} \gamma'_2 \dots \gamma'_{n-1} f_{i_n} \gamma'_n$, $\quad A_{i_k} = (S_{i_k}, F_{i_k})$, $f_{i_k} \in F_{i_k}$, $1 \leq k \leq n$,
- $\{i_1, i_2, \dots, i_n\} = \{1, 2, \dots, n\}$ (every agent works in every derivation step),
- $\gamma_k \stackrel{E}{\Longrightarrow} \gamma'_k$, $\gamma_k \in V^*$, $0 \leq k \leq n$, is the derivation step of the scheme E.

Definition of language

Definition

Let Σ be an 0L eco-colony, $\Sigma = (E, A_1, A_2, \dots, A_n, w_0)$.
The language generated by the type of derivation $x, x \in \{wp, ap\}$ in Σ is

$$L(\Sigma, x) = \{w \in V^* \mid w_0 \overset{x}{\Longrightarrow}{}^{*} w\}$$

Definition

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The language generated by the type of derivation $x, x \in \{wp, ap\}$ in Σ is

$$L(\Sigma, x) = \{w \in T^* \mid w_0 \overset{x}{\Longrightarrow}{}^{*} w\}$$

Example

$\Sigma = (E, A_1, A_2, AbB)$, where
$E = (\{A, B, a, b\}, \{a, b\}, \{a \to a, b \to bb\})$,
$A_1 = (A, \{aB, \varepsilon\})$,
$A_2 = (B, \{aA, \varepsilon\})$

Example

$\Sigma = (E, A_1, A_2, AbB)$, where
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$$AbB \overset{ap}{\Longrightarrow} aBb^2aA \overset{ap}{\Longrightarrow} a^2Ab^4a^2B \overset{ap}{\Longrightarrow} a^3Bb^8a^3A \overset{ap}{\Longrightarrow} \overset{ap}{\Longrightarrow} a^4Ab^{16}a^4B \overset{ap}{\Longrightarrow} a^5Bb^{36}a^5A \overset{ap}{\Longrightarrow} \dots$$

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$\Sigma = (E, A_1, A_2, AbB)$, where
$E = (\{A, B, a, b\}, \{a, b\}, \{a \to a, b \to bb\})$,
$A_1 = (A, \{aB, \varepsilon\})$,
$A_2 = (B, \{aA, \varepsilon\})$

$$AbB \xRightarrow{ap} aBb^2aA \xRightarrow{ap} a^2Ab^4a^2B \xRightarrow{ap} a^3Bb^8a^3A \xRightarrow{ap} \xRightarrow{ap} a^4Ab^{16}a^4B \xRightarrow{ap} a^5Bb^{36}a^5A \xRightarrow{ap} \dots$$

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Generated languages:

$$L(\Sigma, ap) = \{a^n b^{2^n} a^n \mid n \geq 0\}$$
$$L(\Sigma, wp) = \{a^i b^{2^n} a^j \mid n \geq 0,\ 0 \leq i, j \leq n\}$$

Notation

$0EC_x$ $x \in \{wp, ap\}$ class of 0L eco-colonies with x type of derivation

EEC_x $x \in \{wp, ap\}$ class of E0L eco-colonies with x type of derivation

COL_x $x \in \{b, t, wp, sp\}$ class of colonies with the x type of derivation

COL_x^T $x \in \{b, t, wp, sp\}$ class of colonies with the x type of derivation, $V = T$

Results proved in the paper

Theorem

$$0EC_{ap} \subset EEC_{ap} \tag{1}$$

Results proved in the paper

$0EC_{ap} \subset EEC_{ap}$.

The relation $0EC_{ap} \subseteq EEC_{ap}$ is trivial, 0L eco-colonies are special type of E0L eco-colonies (for $V = T$).

$$L_1 = \{a^{2^n} \mid n \geq 1\}$$

This language is generated by the E0L eco-colony
$\Sigma = (E, A_1, A_2, UVa)$,
where $E = (\{a, U, V\}, \{a\}, \{a \to aa, U \to U, V \to V\})$,
$A_1 = (U, \{V, \varepsilon\})$,
$A_2 = (V, \{U, \varepsilon\})$.
$UVa \overset{ap}{\Longrightarrow} VUa^2 \overset{ap}{\Longrightarrow} UVa^4 \overset{ap}{\Longrightarrow} VUa^8 \overset{ap}{\Longrightarrow} UVa^{16} \overset{ap}{\Longrightarrow} \dots \overset{ap}{\Longrightarrow} a^{2^n}$
L_1 is not generated by any 0L eco-colony with *ap* derivation (see the proof of Theorem 1 in the paper). □

Results proved in the paper

Theorem

$$xEC_y - COL_z \neq \emptyset \tag{2}$$

where $x \in \{0, E\}, \; y \in \{wp, ap\}, \; z \in \{b, t, wp, sp\}.$

Results proved in the paper

$xEC_y - COL_z \neq \emptyset$.

We use the language

$$L_2 = \left\{ cda^{2^{2n}} b^{2^{2n}} \mid n \geq 0 \right\} \cup \left\{ dca^{2^{2n+1}} b^{2^{2n+1}} \mid n \geq 0 \right\}$$

This language can be generated by the eco-colony (*ap* as well as *wp*) $\Sigma = (E, A_1, A_2, cda)$,
where $E = (\{a, b, c, d\}, \{a \to aa, b \to bb, c \to c, d \to d\})$,
$A_1 = (c, \{d\})$,
$A_2 = (d, \{c\})$.
$cdab \Rightarrow dca^2b^2 \Rightarrow cda^4b^4 \Rightarrow dca^8b^8 \Rightarrow cda^{16}b^{16} \Rightarrow \dots$
L_2 is not generated by any colony with b, t, wp, sp derivation (see the proof of Theorem 2 in the paper). □

Results proved in the paper

Theorem

$$COL_x - 0EC_{wp} \neq \emptyset, \quad x \in \{b, t, wp, sp\} \tag{3}$$

Results proved in the paper

$COL_x - 0EC_{wp} \neq \emptyset$.

The language

$$L_3 = \{a^{15-2n}b^n cb^n d \mid 0 \leq n < 7, n \text{ is even}\} \cup \{a^{15-2n}b^n db^n c \mid 0 < n \leq 7, n \text{ is odd}\}$$

is a finite language, so $L_3 \in COL_x$ for $x \in \{b, t, wp, sp\}$.

This language is not in $0EC_{wp}$ (see the proof of Theorem 3 in the paper). □

Results proved in the paper

Theorem

$$COL_x - 0EC_{ap} \neq \emptyset, \quad x \in \{b, t, wp, sp\} \tag{4}$$

Results proved in the paper

$COL_x - 0EC_{ap} \neq \emptyset$.

The language

$$L_4 = \{a, aa\}$$

is not generated by any 0L eco-colony. But this language is finite, so $L_4 \in COL_x$ for $x \in \{b, t, wp, sp\}$ (see the proof of Theorem 4 in the paper). □

Results proved in the paper

Theorem

$$COL_b \subset EEC_{ap} \tag{5}$$

Results proved in the paper

Theorem

$$COL_b \subset EEC_{ap} \tag{5}$$

The proof can be found in the paper.
We demonstrate the construction of the proof on the colony generating this language:

$$L_5 = \left\{ waw^R a^i \mid w \in \{0,1\}^*,\ i > 0 \right\}$$

Example: $L_5 = \left\{ waw^R a^i \mid w \in \{0,1\}^*,\ i > 0 \right\}$

Colony: $\mathcal{C} = (\{S, H, H', A, A', 0, 1, a\}, \{0, 1, a\}, \mathcal{R}, S)$
the set of components $\mathcal{R}$ is

$$\mathcal{R} = \left\{ \begin{array}{lll} (S,\ \{HA\}), & (H,\ \{0H'0, 1H'1, a\}), & (A,\ \{aA', a\}), \\ & (H',\ \{H\}), & (A',\ \{A\}) \end{array} \right\}$$

Example: $L_5 = \left\{ waw^R a^i \mid w \in \{0,1\}^*,\ i > 0 \right\}$

Colony: $\mathcal{C} = (\{S, H, H', A, A', 0, 1, a\}, \{0, 1, a\}, \mathcal{R}, S)$
the set of components $\mathcal{R}$ is

$$\mathcal{R} = \{\ (S,\ \{HA\}),\ (H,\ \{0H'0, 1H'1, a\}),\ (A,\ \{aA', a\}),\ (H',\ \{H\}),\ (A',\ \{A\})\ \}$$

Agens work but do not make the word (longer axiom),
components $\Longrightarrow$ rules in the environment.

E0L eco-colony with *ap* derivation: $\Sigma = (E, A_1, A_2, BCS)$,
$E = (\{B, C, S, H, H', A, A', 0, 1, a\},\ \{0, 1, a\},\ P)$,
$A_1 = (B, \{C, \varepsilon\})$, $A_2 = (C, \{B, \varepsilon\})$,
the set of rules P in the environment is

$$P = \{\ H \to H|0H'0|1H'1|A,\ H' \to H'|H,\ 1 \to 1,\ a \to a,\ A \to A|aA'|a,\ A' \to A'|A,\ 0 \to 0,\ S \to S|HA\}$$

Results proved in the paper

Corollaries

1. $$xEC_y - COL_z^T \neq \emptyset$$
   where $x \in \{0, E\}$, $y \in \{wp, ap\}$, $z \in \{b, t, wp, sp\}$.
2. The set of languages $0EC_{wp}$ is incomparable to the sets of languages COL_b, COL_t, COL_{wp} and COL_{sp}.
3. $$COL_b^T \subset EEC_{wp}, \quad COL_{wp}^T \subset EEC_{wp}$$
4. $$COL_b^T \subset EEC_{ap}$$
5. $$COL_b^T \subset EEC_{ap}$$

Follow from the previous results.

Thank you
for your attention.